Crowd motion modeling and evolution PDE with density constraints

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In this talk I will explain a simple model for the movement of the density ρ of a crowd, subject to a non-overcrowding constraint $\rho\leq 1$. This model, inspired by some granular contact models when the crowd is described by a collection of particles, is very similar to the PDE formulation of the Hele-Shaw flow and can be considered as a limit of porous-medium type equations; it requires the density to be advected by a velocity which could be potentially very irregular. I will present the main ideas to prove existence (in particular, using the fact that this equation has the form of a gradient flow in the Wasserstein space of probability measures and using tools from optimal transport), to prove uniqueness (but here some open questions remain) and to simulate the solutions, together with some variants with different applications (bacteria instead of crowds, fluids, strategic populations...). These models have been studied in collaboration with B. Maury (also from Paris-Sud) and his/our collaborators.